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Assessing risk in grade-tonnage curves in a complex copper deposit, northern Brazil, based on an efficient joint simulation of multiple correlated variables

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Risk quantification in grade-tonnage curves is critical for capital investment in mining projects and can be obtained through geostatistical simulations of orebodies. A practical difficulty may arise in multi-element deposits, as the joint modelling of the related attributes using the traditional co-simulation approaches is computationally intensive and may be impractical for use in the industrial environment. This paper presents the construction of risk-integrating grade-tonnage curves for a complex copper deposit in northern Brazil, by jointly simulating its key geochemical attributes of interest: Cu, Fe and K. The joint conditional simulation of these elements is based on Minimum/Maximum Autocorrelation Factors (MAF). MAF is an approach, based on principal components, that spatially decorrelates the variables involved to non-correlated factors. MAF's spatial decorrelation at any lag distance is the main and critical difference of this approach from the principal component approach attempted in the past. In the MAF approach, the independent factors are individually simulated and back-transformed to the conditional simulations of the correlated deposit attributes that reproduce the cross-correlations of the original variables.

Keywords: Joint simulation; mini/max autocorrelation factors; grade-tonnage curves.

Introduction

Capital investment in mining projects requires the quantification, understanding and assessment of risk in grade-tonnage curves. Geostatistical simulation technologies[1] provide an increasingly recognized tool to model geological uncertainty and quantify geological risk associated with grade-tonnage curves. Frequently, mineral deposits and their geological characteristics are described by a multitude of geochemically interrelated attributes. The joint modelling of these attributes assists the geological plausibility of complex orebody models as well as the modelling of individual attributes. A key bottleneck, however, is that the common joint simulation methods[2,3] are too computationally intensive to be of practical use in the industrial environment, particularly when more than two attributes are considered. Contributors to complexity include the tedious inference and modelling of cross-variograms, and computational inefficiency, substantially increasing with the number of variables being co-simulated.

A practical alternative to the 'direct' co-simulation of variables is the decorrelation of variables introduced by David[4]. His approach, demonstrated in the joint simulation of a uranium deposit, is based on the decorrelation of variables using principal component analysis[5–7] (PCA). The effectiveness of this approach, in the presence of spatial cross-correlations inherent in mineral deposits, is limited because PCA ignores cross-correlations at distances other than zero. To overcome the limitations of the 'direct' co-simulation methods and PCA, Desbarats and Dimitrakopoulos[8] have suggested the use of the so-termed Minimum/Maximum Autocorrelation Factors, or MAF, in the context of spatial simulation. The MAF approach may be described as an approach, based on principal components, that spatially decorrelates the variables involved to non-correlated factors. The independent MAF are individually simulated and back-transformed to the conditional simulations of the correlated deposit attributes, that reproduce the cross-correlations of the original variables.

This paper presents the joint simulation of copper, iron and potassium in an oxide copper deposit located in northern Brazil (Figure 1) and the resulting assessment of risk in grade-tonnage curves for copper. The geological environment of the deposit is complex. It occurs within a faulted sequence of Archean felsic-basic metavolcanics hosting hydrothermal breccias and alteration haloes. Copper is dispersed throughout the weathering profile but is also disseminated and enriched in sheet-like saprolitic units outside of the weathered mineralized breccia bodies. The ability of the deposit to supply ore for a SX-EW metallurgical process plant is a key point assessed in a pre-feasibility study, and the joint simulation is particularly important in assessing copper solubility controlled by iron and potassium content.

The following sections include; a summary of the method of joint simulation of multiple correlated variables based on MAF; a description of the deposit and the data available; the results of the joint simulation; the presentation of grade-tonnage curves and risk analysis; and conclusions.

Joint simulation of correlated variables using minimum/maximum autocorrelation factors

In geostatistical terminology, the attributes of a multi-element mineral deposit are represented by a multivariate stationary and ergodic random function. Consider a multivariate, ℓ dimensional, Gaussian, stationary and ergodic spatial random function $Z(x) = [Z_1(x),...,Z_\ell(x)]^T$. The Minimum/Maximum Autocorrelations Factors are defined as the ℓ orthogonal linear combinations $Y_i(x) = a_i^T Z(x)$, $i = 1,.., \ell$ of the original multivariate vector $Z(x)$. MAF are derived assuming that Z (x) is represented by a two-structure linear model of co-regionalization[6]. The MAF transformation can be rewritten as

$$Y(x) = A_{MAF} Z(x) \qquad [1]$$

and the MAF factors are derived from

$$A_{MAF} = Q_2 \Lambda_1^{-1} Q_1 \qquad [2]$$

where the eigenvectors Q_1 and eigenvalues Λ_1 are obtained from the spectral decomposition of the multivariate covariance matrix B of $Z(x)$ at zero lag distance. More specifically,

$$Q_1 B Q_1^T = \Lambda_1 \qquad [3]$$

and Q_2 is the matrix of eigenvectors from the spectral decomposition

$$Q_2 M(\Delta) Q_2^T = Q_2 \left(\frac{1}{2} \left[\left[\Gamma_Y(\Delta) \right]^T + \left[\Gamma_Y(\Delta) \right] \right] \right) Q_2^T \qquad [4]$$

where matrix $\Gamma_Y(\Delta)$ is an asymmetric matrix variogram at lag distance Δ for the regular PCA factors $Y(x)=Z(x)\ A$, where $A = Q\Lambda^{-1/2}$. In practice, several Δ lag distances may be used for values lower than the range and the resulting eigenvectors averaged.

Given the MAF transformation above, the joint simulation of multiple correlated variables using the MAF approach proceeds as follows:

- Normalize the variables to be simulated
- Use MAF to generate the MAF non-correlated factors
- Produce variograms for each MAF
- Conditionally simulate each MAF using any Gaussian simulation method
- Validate the simulation of factors
- Back-transform simulated MAF to variables and denormalize
- Validate the final results
- Generate additional simulations, as needed.

In most cases, the reblocking of the generated realizations to a block support model is required and may be seen as an additional step in the above algorithm.

The deposit and data available

The copper oxide deposit considered in this study is located in northern Brazil (Figure 1). It is approximately 2 km long and 500 m wide and appears as a prominent NW–SE aligned hill. The copper mineralization is hosted by copper oxide minerals, mainly malachite, in a weathered hydrothermal breccia and in a large weathering cap where copper is disseminated and enriched in sheet-like mineral-rich saprolitic units, as shown schematically in Figure 1 (bottom). Metallurgical studies indicate that iron and potassium are key elements for predicting copper recovery, indicating the need to evaluate all these three elements throughout the deposit. The orebody mineralization model supplied by project staff is a weathering envelope that hosts oxide ore, Cu-oxide rich ore and Cu enriched in saprolite ore, with saprolitic ore superimposed over mainly basic volcanics. The deposit in this study is divided into two units: Sector 11 and Sector 12. The available data include 654 RC and 310 DDH drillholes with samples analysed by ICP-Plasma (Cu-K) and X-ray. There are 1136 five-metre composites available in Sector 11 and 866 in Sector 12. The descriptive summary statistics for Cu, Fe and K composites for Sectors 11 and 12 are given in Table I. Figure 2 shows the corresponding histograms for Sectors 11 and 12, respectively.

Joint simulation of copper, iron and potassium

Normal-score transformation

Following the simulation steps using MAF described earlier, a normal-score transformation is performed on the Cu, Fe and K composites available in Sector 11 and Sector 12. Normal score transformations are based on rank ordering of the data and decrease the influence of outliers. This, in turn, assists the inference of the variogram and estimation of covariance matrixes in the simulation process that follows.

MAF transformation

The transformation matrix A_{MAF} (Equation [1]) used to generate the three min/max autocorrelation factors in Sectors 11 and 12 is shown in Table II. MAF are calculated by multiplying the vector of elements Cu, Fe and K by a vector of loadings from the rows of the transformation matrix. It should be noted that the MAF loadings are quite different from the ones derived by PCA[8]. The lag Δ in Equation [4] used in this example is 20 metres and was derived experimentally by testing several lag distances to assure a suitable decorrelation and stable MAF decomposition. Figure 3 shows examples of cross-variograms between MAF from the present study that demonstrate variable decorrelation. Experimental variograms and cross-variograms for Cu, Fe and K are shown and discussed in more detail in a subsequent section.

Variography of MAF

Variography on each MAF is performed. Figure 4 shows the experimental and model variograms fitted to the three MAF in Sector 11 and Sector 12. Note that all variogram models are spherical. MAF variograms are subsequently used in the simulation of each factor and the validation of the MAF simulation results. MAF variograms show clear spatial patterns, as expected. It should be noted that MAF variograms are linear combinations of the variograms of the original (normal score) variables.

Conditional simulation of MAF

Conditional simulation is performed independently on the three MAF using a sequential algorithm[9] based on the generalized sequential Gaussian simulation method[10]. The simulations are performed on a grid of 320075 nodes within the geological limits of Sector 11 and 239375 nodes within Sector 12. Twenty simulations are generated in this study for each of the two sectors and are validated in detail for reproduction of data, histograms and variograms. The validation of the MAF simulations is not presented here as a subsequent section presents the validation of realizations in the data space.

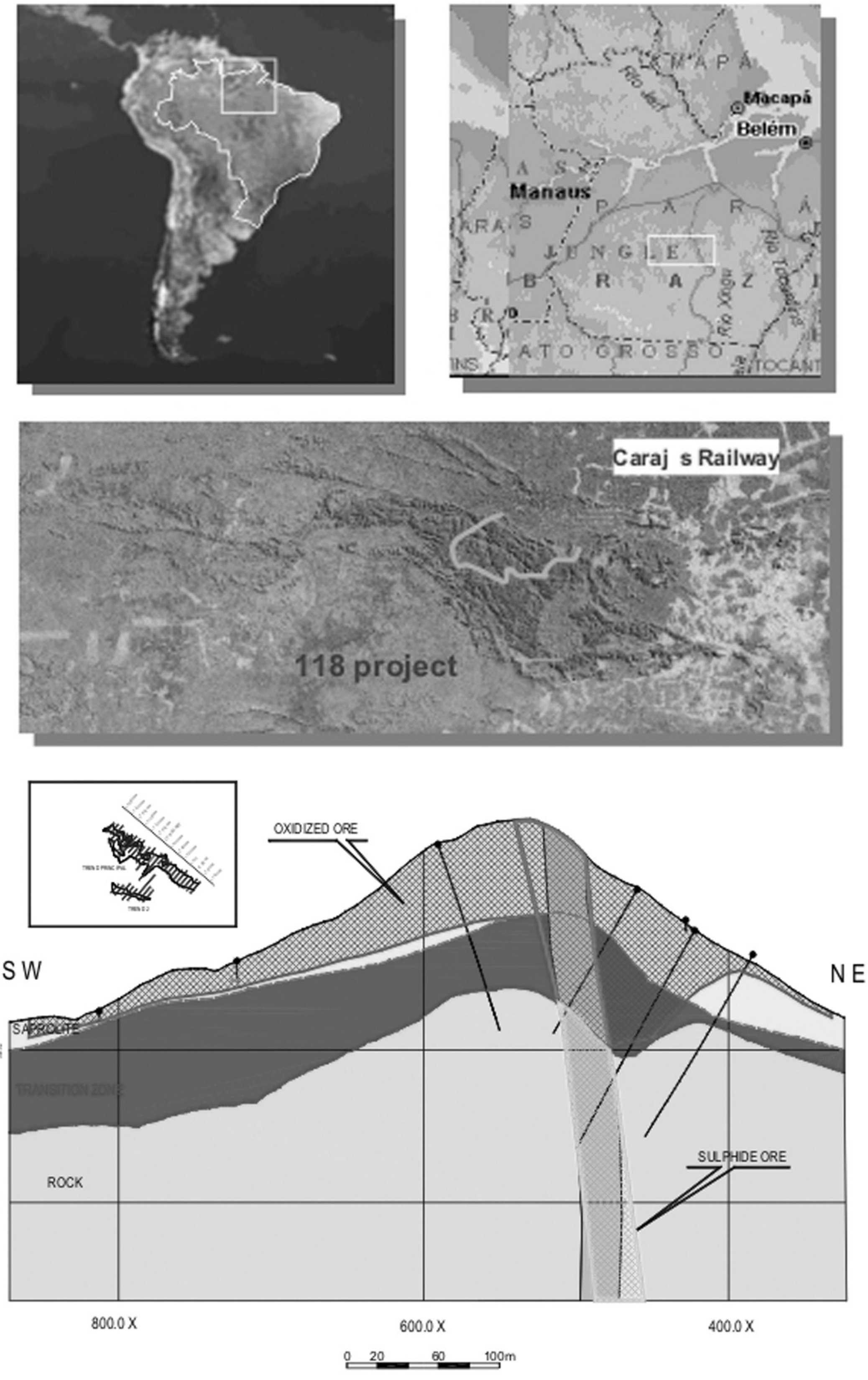

Figure 1. Location map of the project and study area; and a schematic vertical section of the geological model of the deposit

Back transformations of MAF

The realizations of MAF were transformed back to simulated normal score variables by multiplying a column vector of simulated MAF in each grid node with the corresponding inverse matrix of the MAF loadings in Table II. Subsequently, the normal score Cu, Fe and K realizations are back transformed to the data space, and point support realizations are reblocked to $10 \times 10 \times 8$ metre blocks. Figure 5 shows three realizations of Cu through horizontal sections and a vertical section of the deposit.

Table I
Descriptive statistics of 5-metre composites for Sectors 11 and 12

Cu % - Sector 11		Fe % - Sector 11		K ppm—Sector 11	
Mean	0.41	Mean	10.93	Mean	14341.70
Median	0.24	Median	10.86	Median	11111.60
Std Deviation	0.59	Std Deviation	3.77	Std Deviation	14038.68
Kurtosis	75.83	Kurtosis	-0.28	Kurtosis	-0.52
Asymmetry	6.81	Asymmetry	0.29	Asymmetry	0.71
Minimum	0.01	Minimum	2.68	Minimum	123.00
Maximum	9.03	Maximum	25.04	Maximum	60300
Cu % - Sector 12		**Fe % - Sector 12**		**K ppm—Sector 12**	
Mean	0.62	Mean	13.21	Mean	13715.20
Median	0.42	Median	13.70	Median	7679.90
Std Deviation	0.59	Std Deviation	3.65	Std Deviation	15334.67
Kurtosis	20.82	Kurtosis	0.11	Kurtosis	2.32
Asymmetry	2.98	Asymmetry	-0.37	Asymmetry	1.44
Minimum	0.02	Minimum	0.01	Minimum	42.00
Maximum	7.46	Maximum	28.38	Maximum	93528

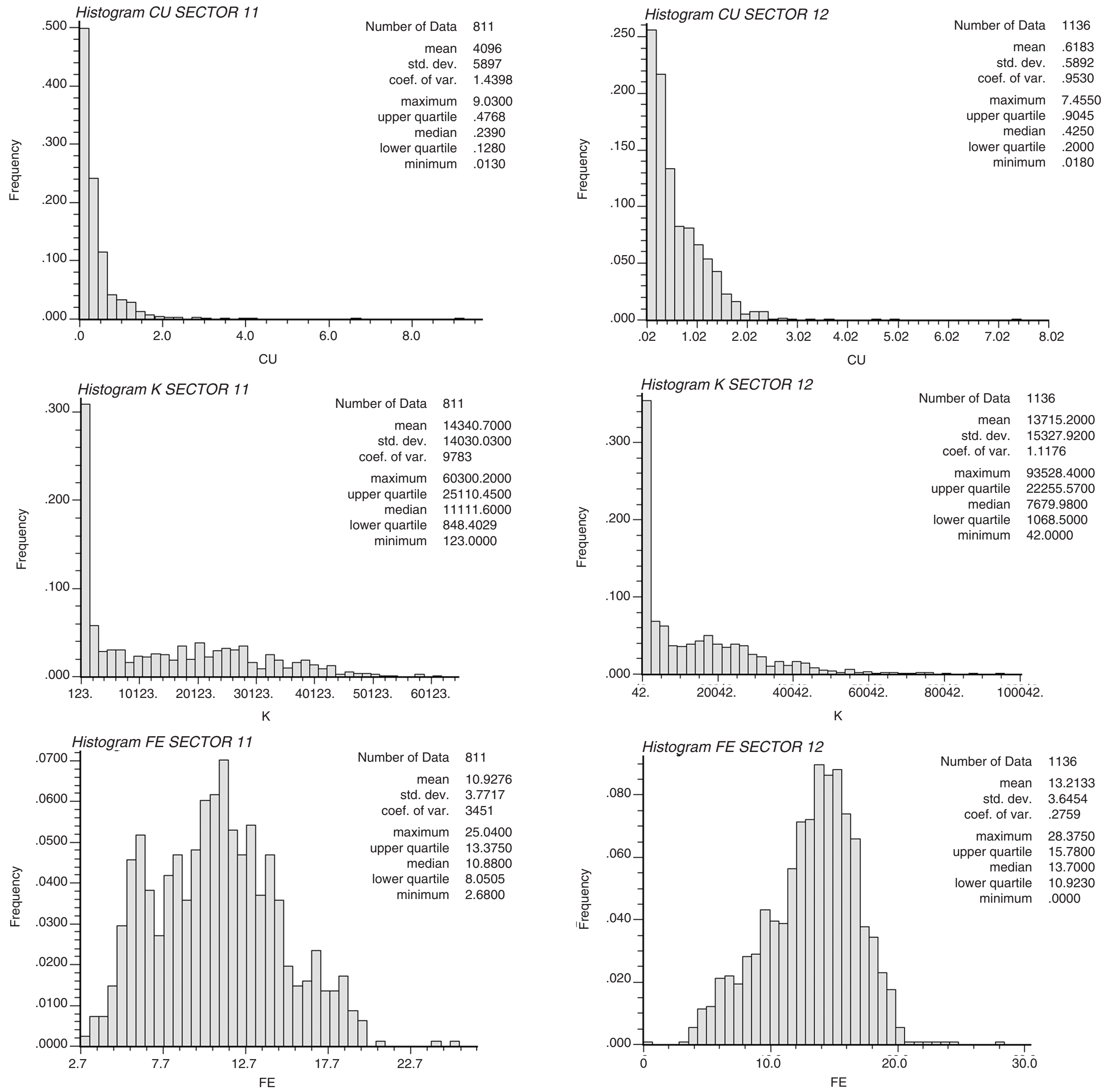

Figure 2. Data histograms of Cu, K and Fe in Sector 11 (left) Sector 12 (right)

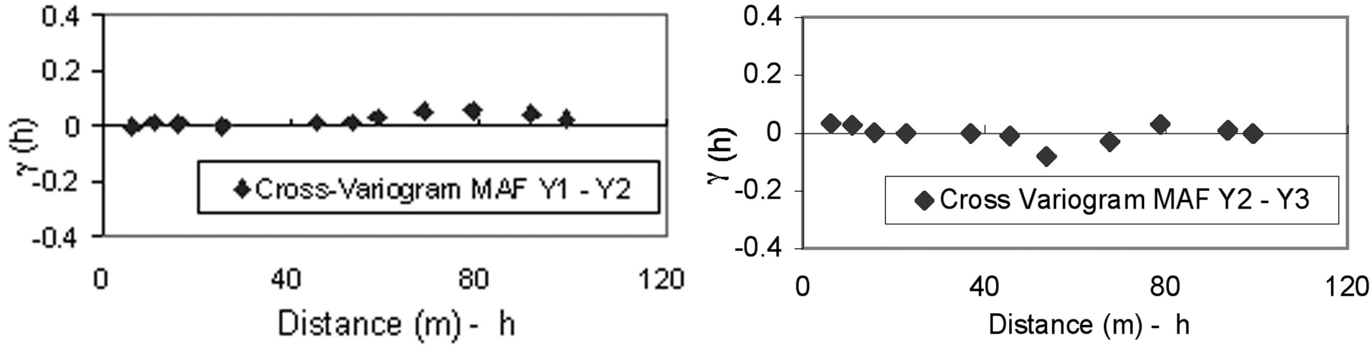

Figure 3. Cross variograms of MAF factors showing decorrelation

Table II
Transformation matrix A_{MAF} for Sectors 11 and 12

Sector 11	-0.930	0.007	-0.126	Sector 12	0.704	-0.633	0.495
	0.363	0.971	-0.400		0.697	0.773	0.008
	0.057	0.238	0.908		-0.135	0.051	0.869

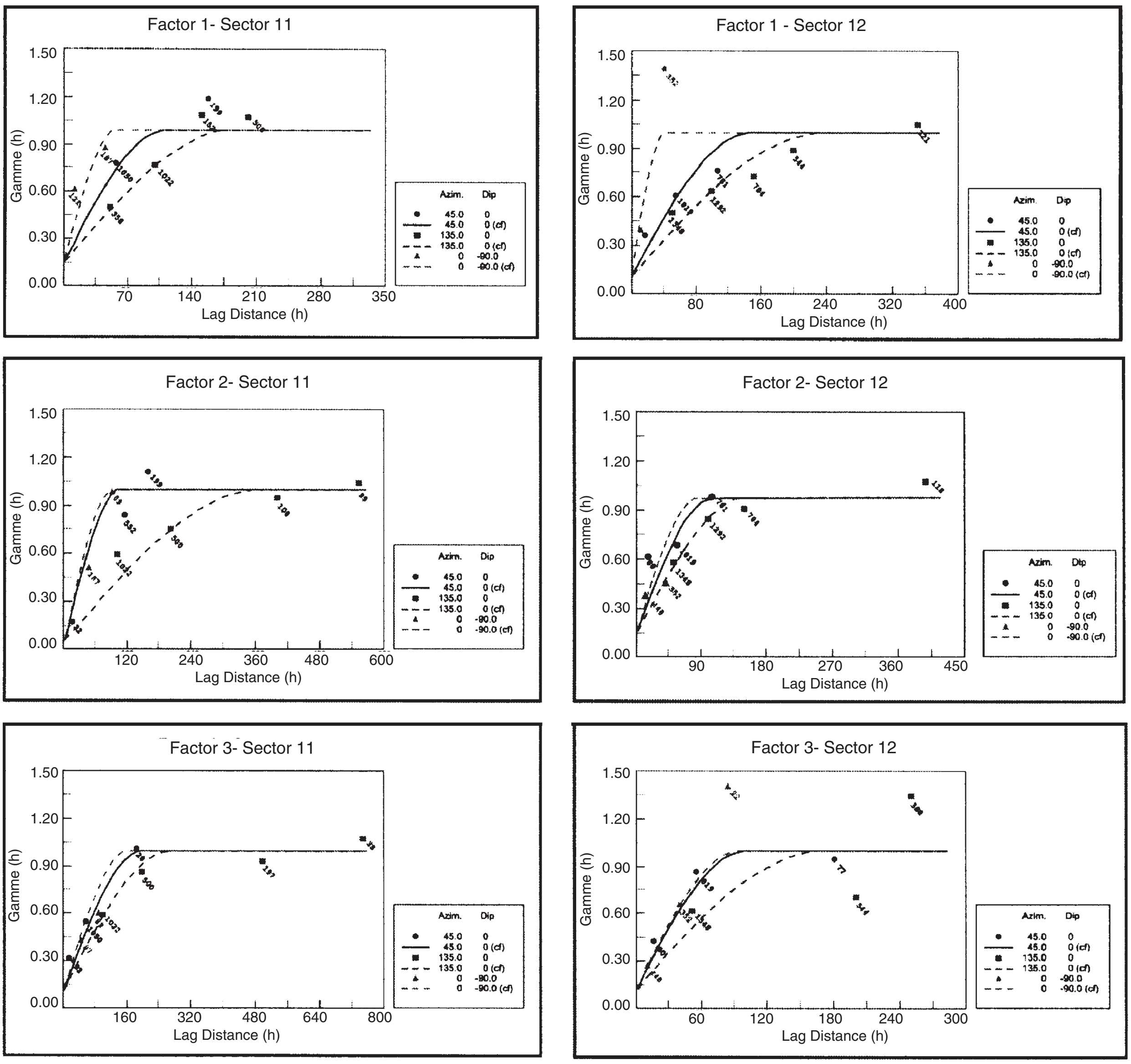

Figure 4. Experimental and model variograms of the three MAF factors in Sector 11 (left column) and Sector 12 (right column)

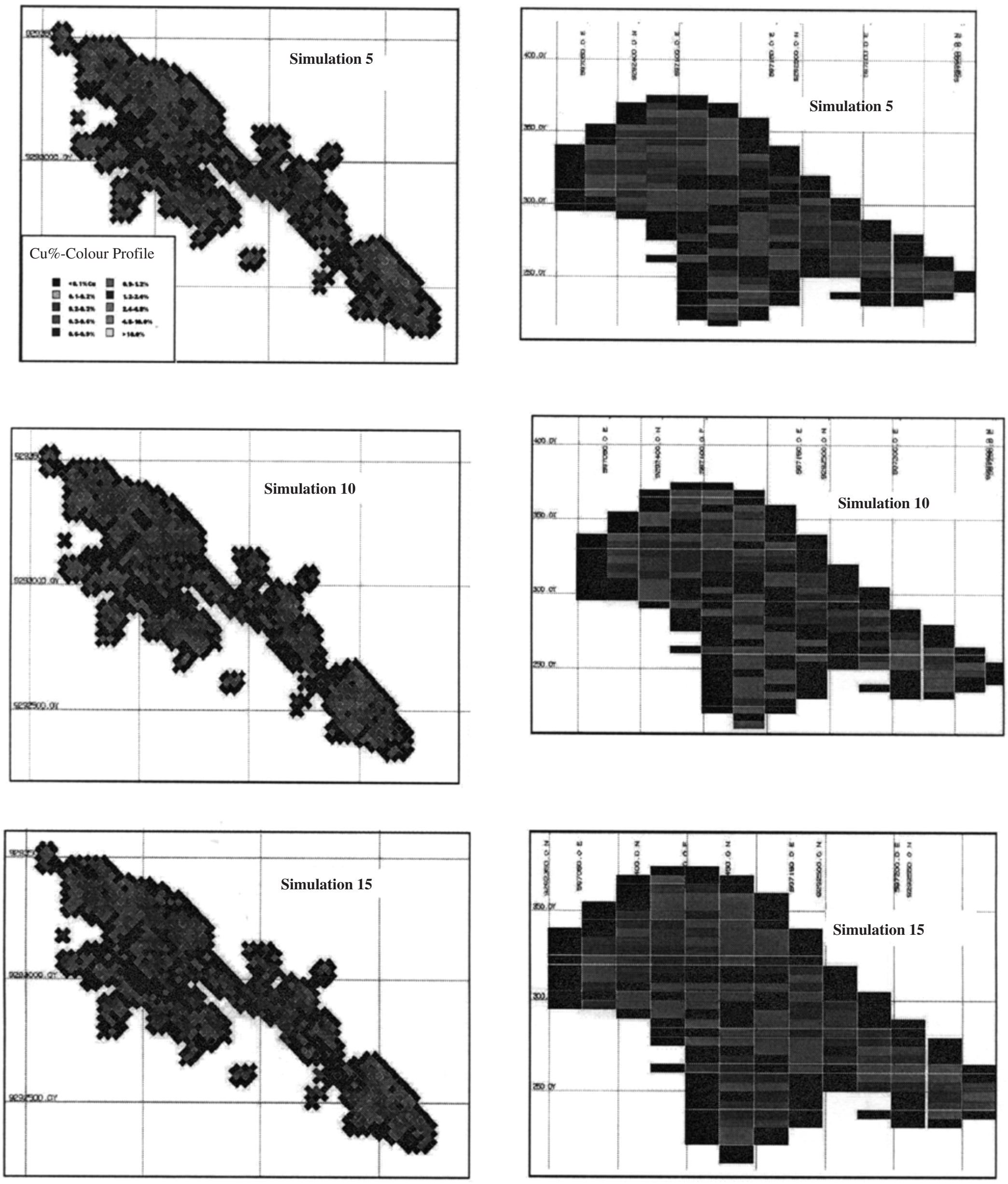

Figure 5. Selected conditional simulations of Cu for Sectors 11 and 12 for level #325 (left column) and cross-section 250NW (right column)

Validation of the joint Cu-Fe-K simulation results

Several validation checks are performed to assess the results of the joint simulations of Cu, Fe and K using the MAF transformations in both Sectors 11 and 12. Validation involves calculation of histograms, experimental variograms and cross-variograms of the simulated realizations to ensure reproduction of original data characteristics.

Figure 6 shows histograms from simulated realizations of each variable in both sectors, and these can be compared to the histograms of original data in Figure 2. Figure 7 shows plots of variograms and cross-variograms for data and simulations in Sector 11. All results suggest that the reproduction of the original data spatial characteristics by simulation is excellent. The reproduction of data variograms and cross-variograms for Sector 12 is also excellent. Recall that the variograms and cross-variograms of original variables are not directly used in the joint simulation based on MAF, which used the variograms of the MAF independent factors.

Risk assessment results and discussion

The mining project considered in this study represents a US$120 million capital investment and aims to operate an SX-EW copper recovery plant using traditional sulphuric heap leach technology. One of the most important factors in the economics of this project is the variability of recoverable copper and the associated risk. Preliminary metallurgical tests, summarized in Table III, indicate that recoverable copper is controlled by the content of iron and potassium, in addition to the original copper grade. This highlights the importance of integrating the geological model of the deposit with jointly simulated orebody scenarios and metallurgical results so as to better identify the possible variability of copper ore output (Cu tonnes) from the SX-EW recovery plant.

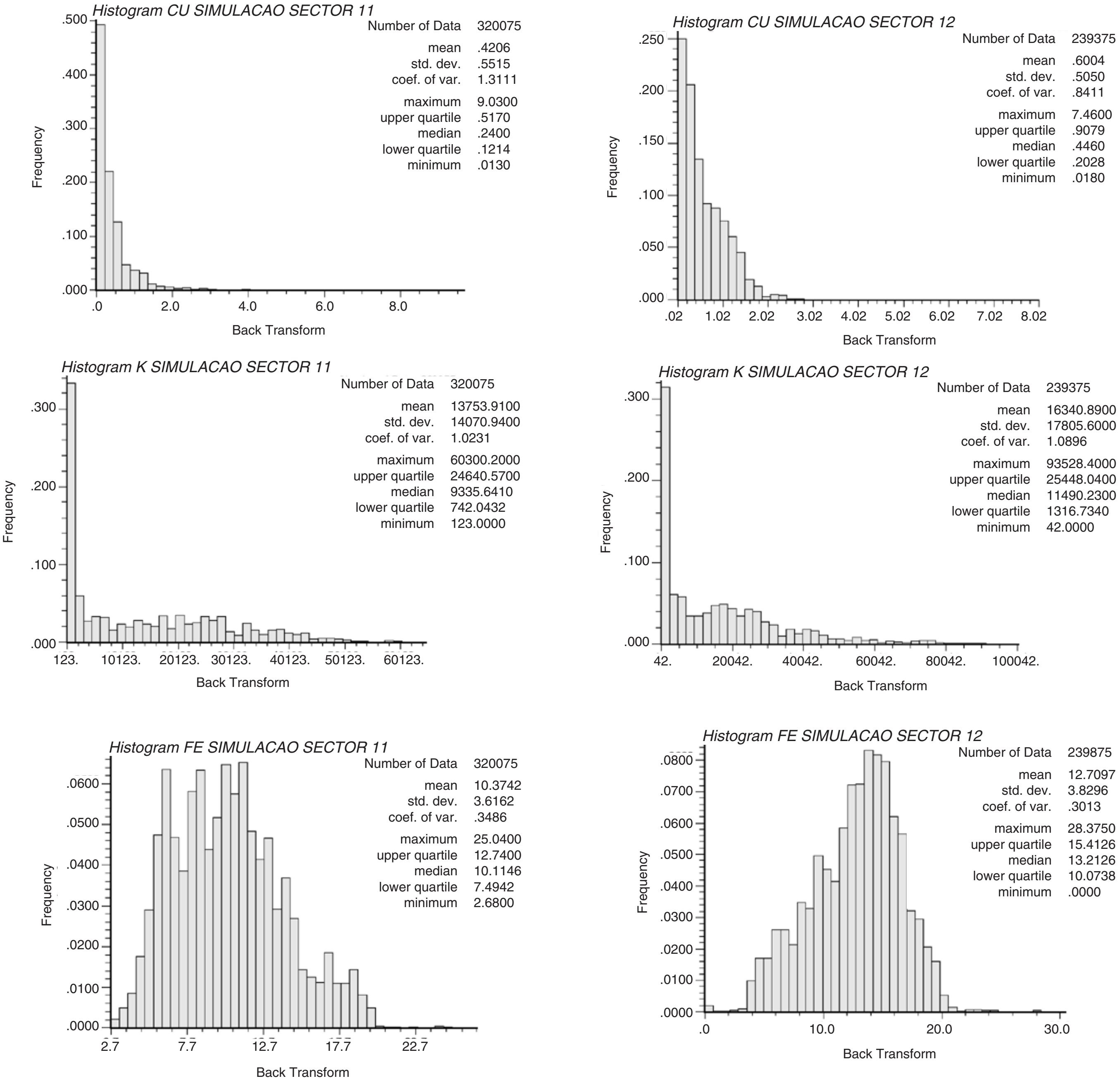

Figure 6. Histograms for joint simulation of Cu, K, Fe using MAF. Sector 11 is shown on the left column and Sector 12 in the right

Given the above reasoning, the assessment of Cu variability and risk in grade-tonnage curves for the deposit is based on recoverable Cu. Recoverable Cu is calculated as a function of the jointly simulated Cu, Fe and K contents for each block of the orebody models and the relationships derived from metallurgical tests, shown in Table IV. The joint simulations described in the previous section provide the information needed to generate realizations of recoverable Cu at the support scale of 10 × 10 × 8 metre blocks, and to subsequently generate grade-tonnage curves for the deposit based on a range of Cu, Fe and K grades. The resulting grade-tonnage curves are then suitable for risk analysis as well as for 'average type' assessments for project economics.

Figures 8, 9 and 10 present the grade-tonnage curves for recoverable Cu as a function of Cu, Fe and K cutoffs. All Figures plot the results from individual simulations and the average (e-type) from the simulations. It is apparent from all grade-tonnage graphs that recoverable copper tonnage variability is more clearly related to the *in situ* copper and potassium content than to iron content. Although there is no restriction to a specific cutoff grade for Fe and K in the metallurgical process, it is clear that for Fe-rich ores no wide variability in copper output is apparent in the grade-tonnage curves. This is supported by the geological model, where Fe-rich grade zones are normally rich in malachite, a mineral with better kinetics in the metallurgical process, compared with iron for acid consumption, and therefore not reducing copper recovery. Variability in recoverable copper grade is, however, mainly dependent on the *in situ* copper content. These results reflect the related metallurgical tests and the recovery functions developed, where the *in situ* copper content is the predominant control on recoverable copper.

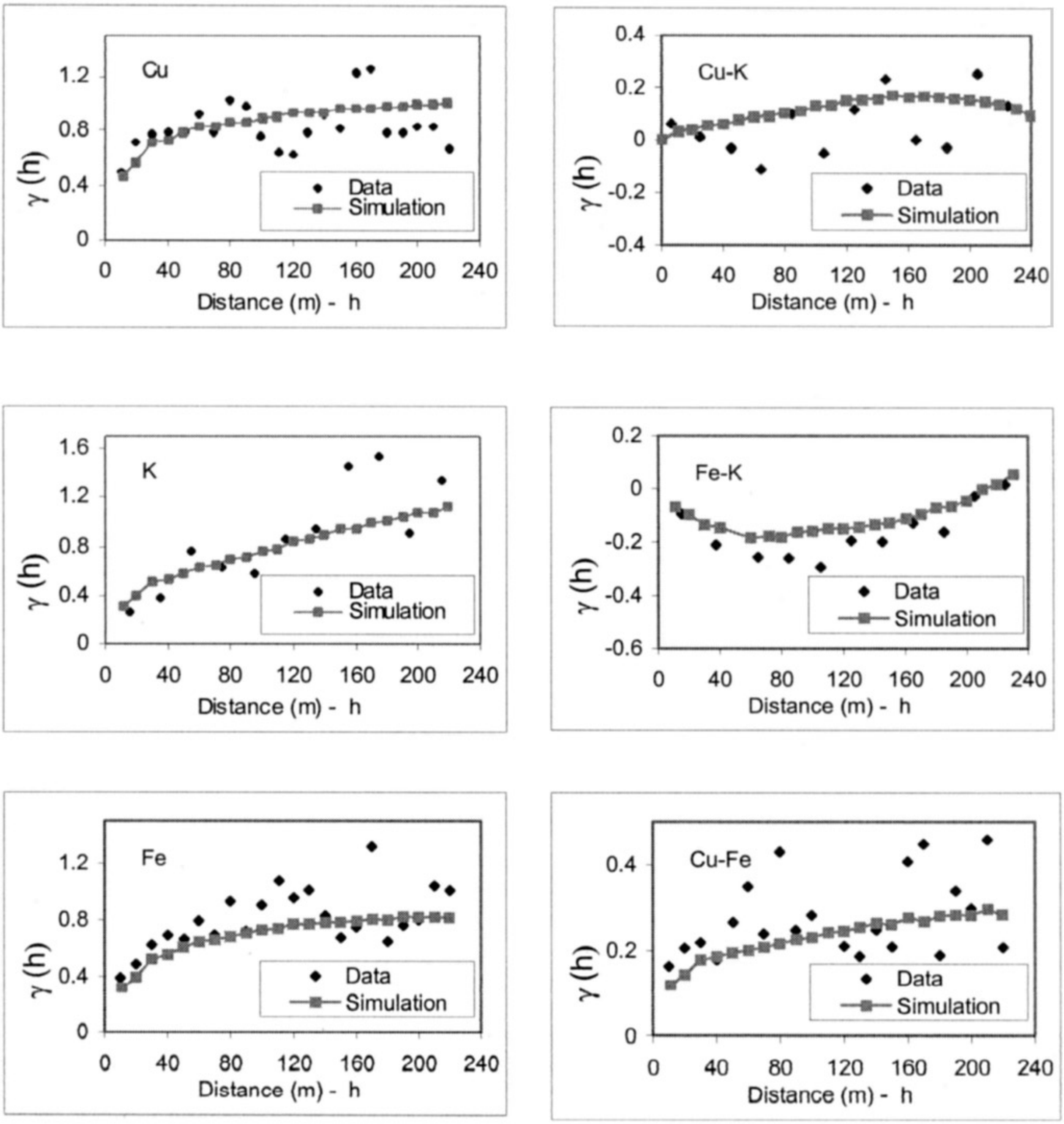

Figure 7. Reproduction of data variograms and cross-variograms of Cu, K, Fe, Cu-K, Fe-K and Cu-Fe in a simulation of Sector 1j note that MAF uses only the variograms of the MAF independent factors

Table III
Preliminary metallurgical Cu recovery tests for the deposit under study

ORE TYPE	NUMBER OF TESTS	AVERAGE GRADE %						Cu% (Rec)	Rec%
		Cu (Head grade)	Al	K	Mg	Fe	Fe_2O_3		
SAPROLITE A	55	0.83	7.3	0.9	1.3	1.0	14.1	0.57	70.02
SAPROLITE B	14	0.28	7.8	1.2	1.0	0.6	16.8	0.09	35.49
SEMI-WEATHERED	9	1.2	5.1	0.4	2.3	4.0	6.43	1.01	84.65
SEMI-WEATHERED A	6	0.45	6.3	1.1	1.7	1.5	12.9	0.28	62.53

Table IV
Recoverable Cu equations from metallurgical tests and for different rock types

SAPROLITE A	Cu Rec (%) = 0.05+0.93*Cu-0.018*Fe+0.007*K
SAPROLITE B	Cu Rec (%) = 0.04+0.62*Cu-0.009*Fe+0.02*K
SEMI-WEATHERED	Cu Rec (%) = 0.04+0.69*Cu-0.030*Fe-0.04*K

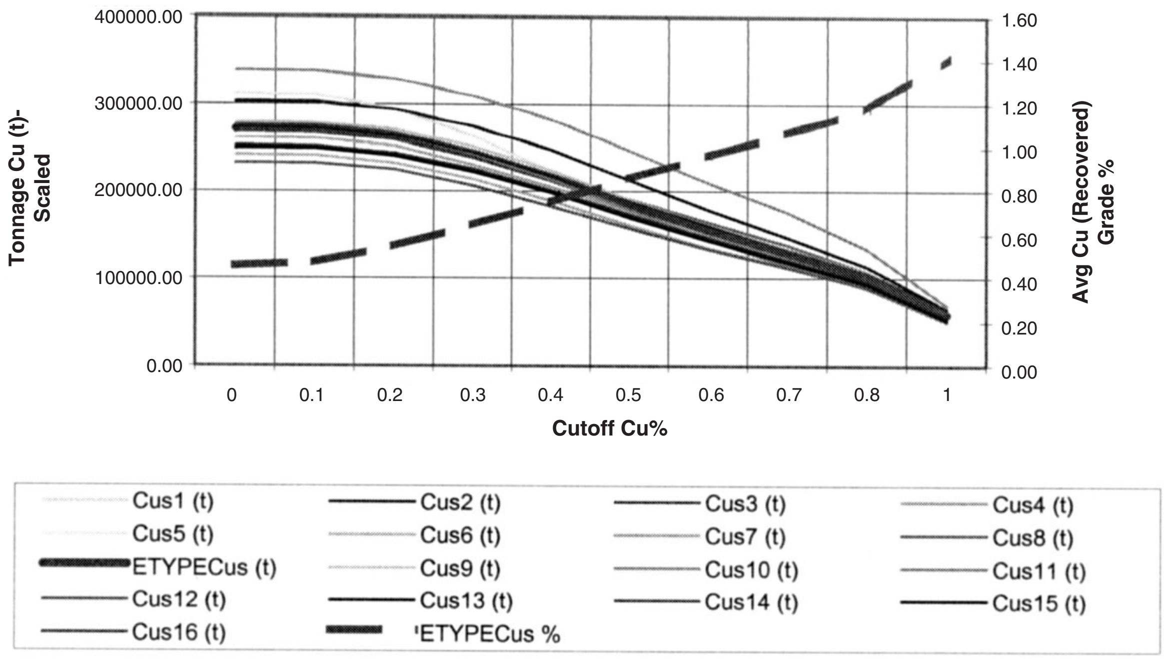

Figure 8. Grade-tonnage curves for recoverable Cu as a function of Cu cutoffs. Thin lines represent individual simulations and thick dashed lines represent the average (e-type) from the simulations

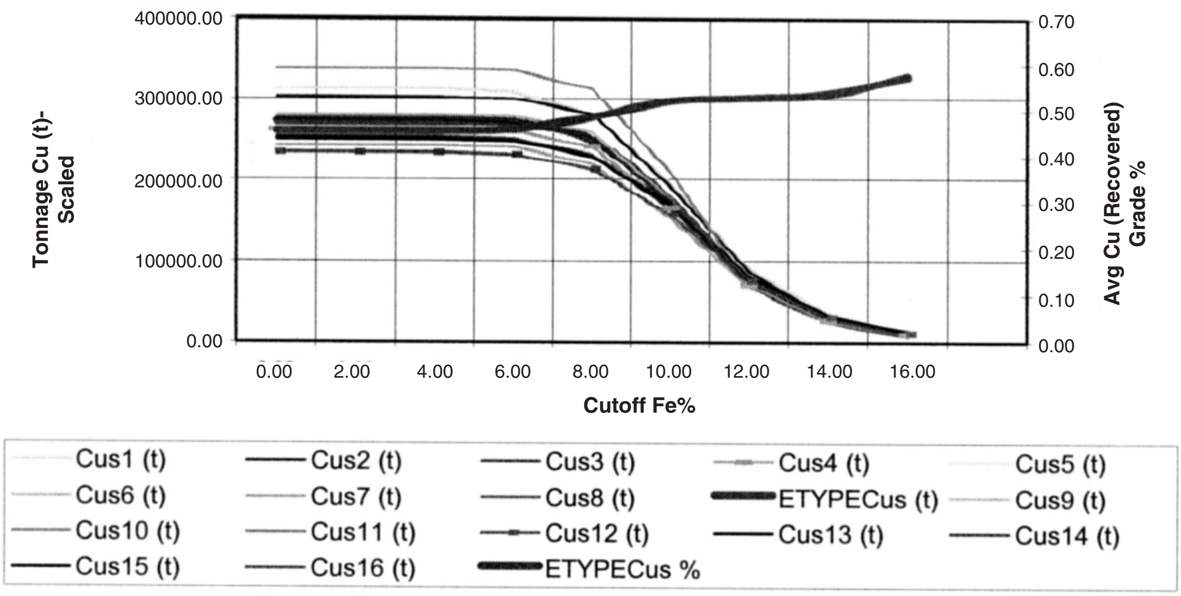

Figure 9. Recoverable copper tonnage as a function of iron content; results from individual simulations and average (e-type) from the simulations

The grade-tonnage curves in Figures 8, 9 and 10, show that the assessment of variability of recovered copper can be investigated by integrating all three elements with the geology of the deposit. This assessment of variability provides a clear description of the inherent uncertainty of potential copper outputs at the plant when analysing cash flows and other financial aspects in the evaluation of the present project. For example, consider a preliminary operational Cu cutoff grade for the project of 0.6% Cu. Figure 8 suggests that the output from the SX-EW recovery process could range from 145000 to 210000 tonnes of copper. With a planned annual Cu production of 45000 tonnes, the results suggest that the potential exists for the project life to be substantially expanded.

At present, no restrictions to any specific cutoff grades for Fe and K are considered in the evaluation of the deposit. However, it is clear from Figure 9 that for the Fe-rich ores, which are mainly composed of hematite and goethite oxidized breccia zones, there does not appear to be wide variability in recoverable copper outputs in the grade-tonnage curves. This may result from the regular occurrence of malachite with Fe-rich ores, which has better kinetics in the metallurgical process, compared with iron, for acid consumption.

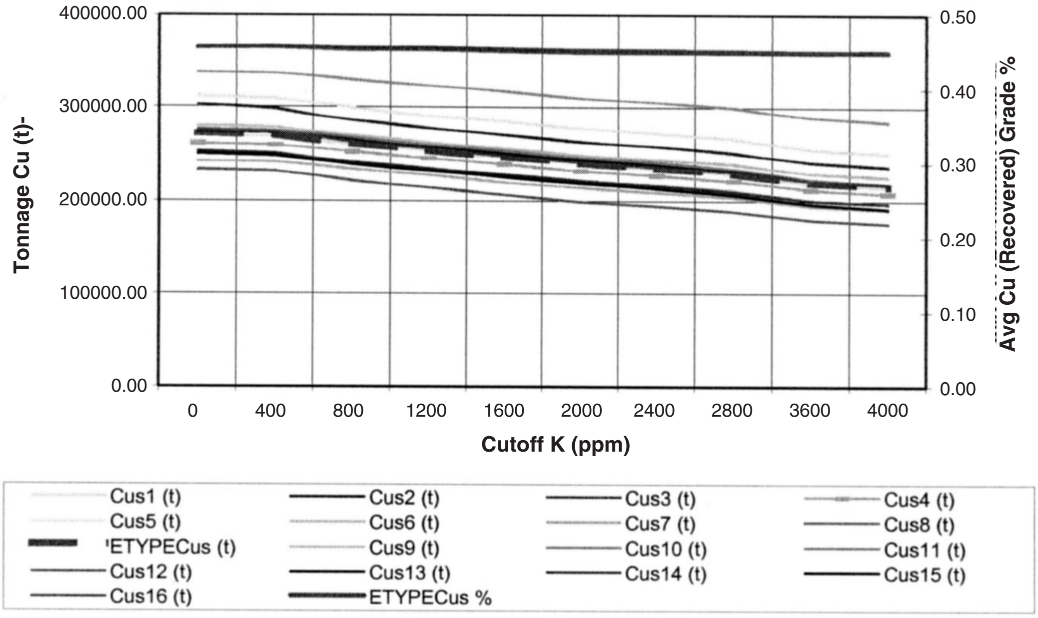

Figure 10. Recoverable copper tonnage as a function of potassium content; results from individual simulations and average (e-type) from the simulations

Conclusions

Risk analysis in grade-tonnage curves for a complex multi-element copper deposit in northern Brazil, or for any multi-element deposit, being considered in a feasibility study is critical for capital investment. This study presented the use of an efficient joint simulation approach based on min/max autocorrelation factors. The approach simplifies joint simulations in mineral deposits by decorrelating variables to independent factors that are then simulated independently. The case study presented herein shows the key practical aspects of the MAF approach and the excellent validation of the results it generates.

The case study has presented three developments: (i) a methodology for investigating uncertainty via jointly simulating Cu, Fe and K; (ii) the construction of recoverable Cu models as a function of solubility, given the Fe and K content and the simulated realizations of the deposit; and (iii) the quantification of related risk and range of outcomes in Cu grade-tonnage curves. As a result, valuable information is generated for the further analysis of Cu grade and tonnage, solubility and recoverability effects on the appraisal of the project.

Acknowledgements

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